On the Dynamic Colouring of Generalised Mycielskian of some graph classes

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Abstract

Graph colouring has often been a topic of interest because of its wide applications. Among that, Dynamic Colouring of graphs is a more strengthened version which allows us diverse connections rather than all links of the same type. A dynamic colouring of a graph G is a proper colouring of the vertex set $V(G)$ such that for each vertex of degree at least 2, its neighbors receive at least two distinct colours. The smallest integer k such that G has a dynamic k-colouring is called the dynamic chromatic number $\chi_d(G)$ of G [9]. In [7] K.Kaliraj et al. have studied the dynamic chromatic number of cartesian product of complete graph with complete graphs, complete graphs with complete bipartite graphs and wheel graph with complete graphs. A few other works on dynamic chromatic number can be seen in [1, 2, 3].

Mycielskian and Generalised Mycielskian are two interesting graph operators which often help in studying complex problems and advance the research in different areas. For a graph $G = (V, E)$, the Mycielskian of G is the graph $\mu(G)$ with vertex set $V \cup V' \cup \{u\}$, where $V' = \{x' : x \in V\}$ and edge set $E \cup \{x'y : xy \in E\} \cup \{x'u : x' \in V'\}$. The vertex x' is called the twin of the vertex x (and x the twin of x'), and the vertex u is called the root of $\mu(G)$ [4]. Generalised Mycielskian is the extended version of Mycielskian which is defined as follows. Let G be a graph with vertex set $V^0 = \{v_1^0, v_2^0, \ldots, v_n^0\}$ and edge set E^0. Given an integer $m \geq 1$ the $m - Mycielskian$ of G, denoted by $\mu_m(G)$, is the graph $\mu_m(G)$ with vertex set $V^0 \cup V^1 \cup V^2 \cup \ldots \cup V^m \cup \{u\}$, where $V^i = \{v_j^i : v_j^0 \in V^0\}$ is the i^{th} distinct copy of V^0 for $i = 1, 2, \ldots m$, and edge set $E^0 \cup (\bigcup_{i=0}^{m-1} \{v_j^i v_{j'}^{i+1} : v_j^0 v_{j'}^0 \in E^0\}) \cup \{v_j^m u : v_j^m \in V^m\}$ [8]. The colouring of Mycielskian and Generalised Mycielskian are well-studied and, a few works in this direction can be seen in [5, 6, 10]. It is found interesting to connect Mycielskian and dynamic chromatic numbers and in [11] J.V.Vivin et al. have studied

the dynamic chromatic number of Mycielskian of paths and cycles. Motivated by these works, we generalised the results obtained in [11] and also obtained the dynamic chromatic number of the Generalized Mycielskian of Wheels and Complete bipartite graphs.

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